DOCUMENT RESUME

ED 127 206 SE 021 322

AUTHOR Suydam, Marilyn, Comp.
TITLE Electronic Hand Calculators: The Implications for Pre-College Education. Final Report. Abbreviated Version.
INSTITUTION National Science Foundation, Washington, D.C.
PUB DATE Feb 76
GRANT NSF-EPP-75-16157
NOTE 50p.; For related document, see SE 021 321

EDRS PRICE MF-$0.83 HC-$2.06 Plus Postage.
DESCRIPTORS Basic Skills; Curriculum; *Educational Change; Elementary School Mathematics; Elementary Secondary Education; Instruction; Instructional Aids; *Mathematics Education; *Research; Secondary School Mathematics; *Surveys
IDENTIFIERS *Electronic Hand Calculators; National Science Foundation; NSF

ABSTRACT

This volume reports research conducted to provide the National Science Foundation (NSF) with information concerning the existing range of beliefs and opinions about the impact of the hand-held calculator on pre-college educational practice. A literature search and several surveys of groups of individuals involved in calculator manufacture and sales or in education were conducted. In addition, four position papers by prominent educators were obtained. The body of the report presents summaries of information collected relevant to availability of calculators, arguments for and against their use in schools, ways in which they are now used in schools, and research findings. Five recommendations related to study of and preparation for implementation of calculator use are made. A list of references is provided. (SD)

****
* Documents acquired by ERIC include many informal unpublished *
* materials not available from other sources. ERIC makes every effort *
* to obtain the best copy available. Nevertheless, items of marginal *
* reproducibility are often encountered and this affects the quality *
* of the microfiche and hardcopy reproductions ERIC makes available *
* via the ERIC Document Reproduction Service (EDRS). EDRS is not *
* responsible for the quality of the original document. Reproductions *
* supplied by EDRS are the best that can be made from the original. *
****

ED127206

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

Electronic Hand Calculators:

The Implications for Pre-College Education

Final Report

prepared for

The National Science Foundation

Grant No. EPP 75-16157

February 1976

Project Director: Marilyn N. Suydam, The Ohio State University

This report was prepared pursuant to a grant from the National Science Foundation (Grant No. EPP 75-16157). The opinions expressed herein do not necessarily reflect the position or policy of the National Science Foundation.

SE 021 332

Electronic Hand Calculators:

The Implications for Pre-College Education

Table of Contents

	Page
Foreword	iii
I. Introduction	1
Explanation of the Study	1
Procedures Used in the Study: Further Explication	3
II. Availability of Calculators	7
Sales	7
Prices	9
Current Level of Use in Schools	11
Estimates of Future Use	11
III. The Cases For and Against Using Calculators in Schools	12
Arguments For Using Calculators	14
Arguments Against Using Calculators	16
Discussion of Arguments	18
Potential Implications of the Widespread Use of Calculators	19
Summary	23
IV. Ways in Which Calculators Are Now Used in Schools	24
V. Empirical Evidence	27
Comments on Research Reports on Hand-held Calculators	32
Needed Research	37
Summary	37
VI. Some Recommendations	39
Recommendations from Educators Surveyed	39
Major Recommendations	41
Appendix A: Annotated List of References	A-1
Appendix B: Questionnaires and Summaries of Data	
1. From Manufacturers and Distributors	B-1
2. From State Supervisors of Mathematics	B-17
3. From School Personnel	B-27
4. From Teacher Educators	B-38
5. From Textbook Publishers	B-74

		Page
Appendix C:	Teaching Mathematics with the Hand-Held Calculator Position paper prepared by George Immerzeel, Earl Ockenga, and John Tarr	C-1
Appendix D:	Hand-Held Calculators and Potential Redesign of the School Mathematics Curriculum Position paper prepared by H. O. Pollak	D-1
Appendix E:	Some Suggestions for Needed Research on the Role of the Hand-Held Electronic Calculator in Relation to School Mathematics Curricula Position paper prepared by J. F. Weaver	E-1
Appendix F:	Calculators and School Arithmetic: Some Perspectives Position paper prepared by Zalman Usiskin and Max Bell	F-1

Foreword

The information for this report was largely collected in May, June, and July 1975; the Interim Report (originally intended to be the Final Report) was prepared in August 1975. The present paper is an extension, based on the original data and an analysis of what has occurred since the Interim Report was prepared, but without further data collection.

Richard K. Shumway participated in the project and was responsible for developing significant portions of the Interim Report. Because the Project Ending Date was extended without further funding, other professional commitments kept him from participating in the preparation of this Final Report, and he requested that his name not be used in a co-author sense. Nevertheless, many sections of this Report are directly attributable to his earlier analysis: I have attempted to indicate these in the Report. I would also like to express my appreciation for his help.

Marilyn N. Suydam
The Ohio State University

Appreciation is also extended to others who helped in the preparation of the Interim Report: Suzanne Damarin and Paul Wozniak, doctoral students at The Ohio State University, for their assistance in collecting and collating information; Louraine Wagner and Beverly Brooks, for typing project materials and managing certain aspects of the project; colleagues at the University and personnel at NSF for their helpful suggestions -- and most of all, to all those throughout the country who so willingly shared information.

Electronic Hand Calculators:
The Implications for Pre-College Education

I. Introduction

Explanation of the Study

There have been many misconceptions about this project. Therefore, it seems appropriate that this report should begin by stating three things that this project was not designed to do:

- It was not designed to compare the reactions or beliefs of different groups about hand-held* calculators.
- It was not designed to collect specific uses of calculators in the classroom.
- It was not designed to be the precursor of a development project (much as that may be needed).

It was designed to provide a report to The National Science Foundation on the range of beliefs and reactions about calculators, and in particular on the arguments that were being used to support positions strongly favorable and strongly negative toward the use of calculators in elementary and secondary schools. From various sources, an analysis of the status of the calculator was to be developed.

In connection with this:

(1) A restricted questionnaire survey was conducted: the sampling was by design nationwide, but it was not random. Therefore, the results cannot be statistically compared for the purpose of

---

* Hereafter, the word calculator will be used to refer to the hand-held calculator, also termed the electronic calculator, the mini calculcutor (not in the computer sense), or the pocket calculator.

generalizing. The questions were designed simply to elicit the range of reactions, to aid in identifying the arguments being used.

(2) The literature was surveyed to identify what was being said in writing to argue for or against using calculators.

(3) Manufacturers and distributors were surveyed in an attempt to secure information on the current and future status of sales and development.

(4) Experiential reports and research reports were scrutinized to ascertain what has already been learned about how to use calculators and the effect of using calculators.

(5) Telephoning to follow up on information, attending meetings and workshops (local, state, regional, and national) to learn what was said by speakers and audiences, talking with teachers, and similar activities were also part of the process of securing information.

(6) Position papers were requested on several topics, to provide in-depth, thoughtful statements about various aspects related to the use of calculators.

Thus, the original intent of this project was simply:

(1) To collect information regarding the use or non-use of calculators, and to form a <u>list</u> of the reasons why educators and others believed that the calculator should be used in schools or why the calculator should be banned in schools;

(2) To analyze the arguments reported by those questioned and in the literature, in order to determine the potential impact or lack of impact of the calculator on the curriculum; and

(3) To develop a critical analysis of what has and has not been

done with calculators at pre-college levels, what knowledge is or is not available about them, and what implications this has for education at the pre-college level.

Procedures Used in the Study: Further Explication

The steps taken to meet the purposes of the study are described in more detail in this section.

(1) Collect information

Information was collected in three ways: by means of questionnaires; by searching the literature; and by attending meetings and conferences.

(a) Questionnaires

Nine target audiences were identified: calculator manufacturers; supply companies selling calculators; other marketing outlets; state supervisors of mathematics; school districts using and/or studying the use of calculators, not using calculators, or banning the use of calculators; mathematics teacher educators; decision-makers in both elementary and secondary schools (including teachers, administrators, and supervisors); publishers of elementary- and secondary-school textbooks; and curriculum developers.

As the questionnaires to be sent to these groups were being prepared, it became evident that these audiences were not all distinct in their composition. Therefore, five questionnaires were developed, to be sent to: calculator manufacturers and distributors, state supervisors of mathematics, school personnel, teacher educators, and textbook publishers. (These questionnaires are contained in Appendix B.)

Certain questions or points were duplicated on two or more questionnaires, but in general the questions asked of one group differed from those asked another group. The intent was to secure a range of "pro" and "con" answers, rather than to compare the responses of one group with the responses of another. In some cases, telephone contacts were made to secure basic or additional information; in most cases, the questionnaires were sent and returned by mail.

The samples were identified in various ways:

- Calculator manufacturers and distributors were determined primarily by perusal of the literature and advertisements.

All those identified (N = 39) were sent a questionnaire; responses were received from only 7. In an attempt to secure further information, some were contacted by telephone. In addition, a "blind" request for advertising information was sent, and information on specifications of various calculators was collated from these materials.

- Marketing outlets in 20 selected cities were identified and a sample contacted by telephone. However, the sampled outlets indicated that they were unable to provide the type of information requested. Some similar information was, however, secured from marketing journals.

- A list of all those responsible for supervising mathematics in state departments of education was secured, and each person was sent a questionnaire (N = 86, plus 13 Canadian supervisors). Responses were received from 65 persons in 33 states and several provinces.

- School districts and personnel involved in using calculators or taking a position on banning calculators were identified by such procedures as scanning news reports, contacting state supervisors, and querying teacher educators, in addition to contacts made through meetings and conferences. Each person identified was sent a questionnaire (N = 58). Responses were received from 16 teachers and 16 other school personnel, in 20 states.

- Teacher educators who were in a position to make substantive statements about the implications of using calculators were identified primarily through published membership lists. An attempt was made to send a questionnaire to at least one person in each state, as well as to all major curriculum developers. Of 87 questionnaires sent, 78 were returned, from 39 states.

- All publishers of elementary- and secondary-school mathematics textbooks (N = 26) were sent a questionnaire. Responses were received from 13.

(b) Literature

Literature from the following sources was compiled and reviewed:

- Reports from calculator manufacturers and distributors.

- Curriculum materials in ERIC and from other sources.

- Other reports in ERIC (e.g., opinion papers).

- Newspapers.

- Educational journals listed in ERIC's Current Index to Journals in Education.

- Non-education journals (including "popular" and "news" magazines.

- Research, including journal-published reports, dissertations, and other reports and monographs.

- Position papers, conference reports, and other documents.

(c) Meetings

Meetings which were attended included ones in Mt. Clemens, Michigan; Canton, Ohio; Washington, D.C.; Denver, Colorado; and several in Columbus, Ohio. At these meetings there was opportunity to listen to presentations, attend workshops, discuss with teachers and other educators, and review the displays of manufacturers.

(2) Collate, analyze, and synthesize the information collected.

This second stage of the project was the necessary prelude to the development of this report. Questionnaire responses were collated, literature reports were abstracted, and information from meetings and discussions was culled. Appendix A contains an annotated list of most of the literature surveyed. (It should be noted that not all of the articles and reports deal with hand-held calculators: some refer to desk calculators. It was felt that, in view of the limited amount in print on hand-held calculators plus the similarity in use of the two types of calculators, there might be some information on desk calculators which is applicable to hand-held calculators.)

Appendix B contains the complete sets of responses from questionnaires, organized by type of respondee and by question.

(3) Prepare report.

The remainder of the body of this document is the analysis of the above information. This is based on the data collected during May, June, and July 1975, which was incorporated into the Interim Report, plus an analysis of what has occurred since the Interim Report was prepared, but without further data collection. A summary is presented of the most meaningful information on:

- the availability of calculators,

- the status of the case against using calculators,

- the status of the case for using calculators,

- what is going on in schools, and

- empirical evidence.

In the summary are specific recommendations related to the use of calculators.

[Perhaps it should be reiterated that there is no statistical analysis of the responses to the information obtained from the questionnaires. The samples were not randomly selected; the sampling techniques employed were designed only to identify arguments which are being proposed for and against the use of calculators in schools. No effort was made to sample representatively to estimate the extent to which any group holds a certain position or the proportion who hold a position for a certain reason. That this type of survey should not be a goal of the project was stipulated by National Science Foundation personnel early in the development of the proposal.]

In addition to the analysis in the body of the Final Report, several position papers were prepared by persons who have devoted much thought and effort to the promises and problems posed by the use of calculators in the elementary and secondary schools of this country. The topics were identified by a small group of persons meeting in Washington, D.C. on 31 July-1 August 1975. The position papers are incorporated as Appendices C, D, E, and F:

- Appendix C: Teaching Mathematics with the Hand-Held Calculator

  by George Immerzeel, Earl Ockenga, and John Tarr

- Appendix D: Hand-Held Calculators and Potential Redesign of the School Mathematics Curriculum

  by H. O. Pollak

- Appendix E: Some Suggestions for Needed Research on the Role of the Hand-Held Electronic Calculator in Relation to School Mathematics Curricula

  by J. F. Weaver

- Appendix F: Calculators and School Arithmetic: Some Perspectives

  by Zalman Usiskin and Max Bell

II. Availability of Calculators

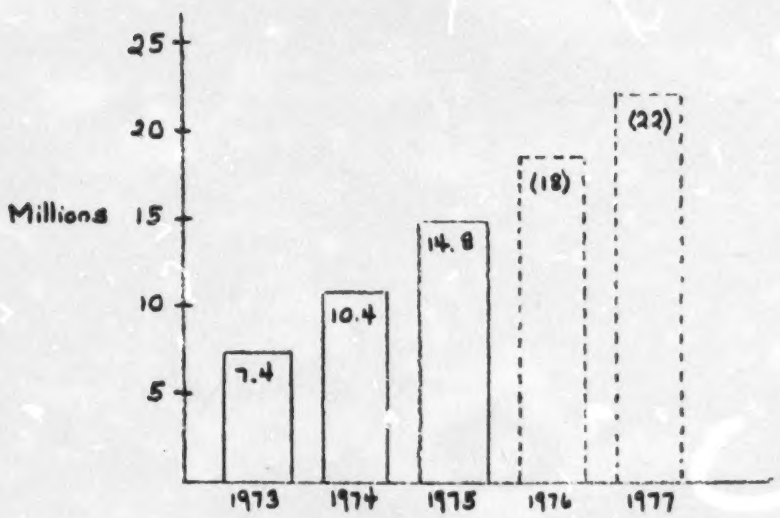
During 1975, it was evident that calculators were "selling like hotcakes". As prices tumbled early in the year, calculators lost their position as status symbols, and reached the point of being considered necessities by many. The level of sales and the implications of data projections for the schools are considered in this section.

Sales

Calculator manufacturers (with one exception) were, to say the least, reluctant to disclose information on sales. Distributors indicated inability to access this information readily. Therefore, much of the information on sales comes from published sources: marketing journals such as Discount Store News, Merchandizing Week, and Electronics regularly publish both data and projections.

For the Interim Report on this project, Shumway developed two tables to summarize such information on sales; one is on sales estimates per year:

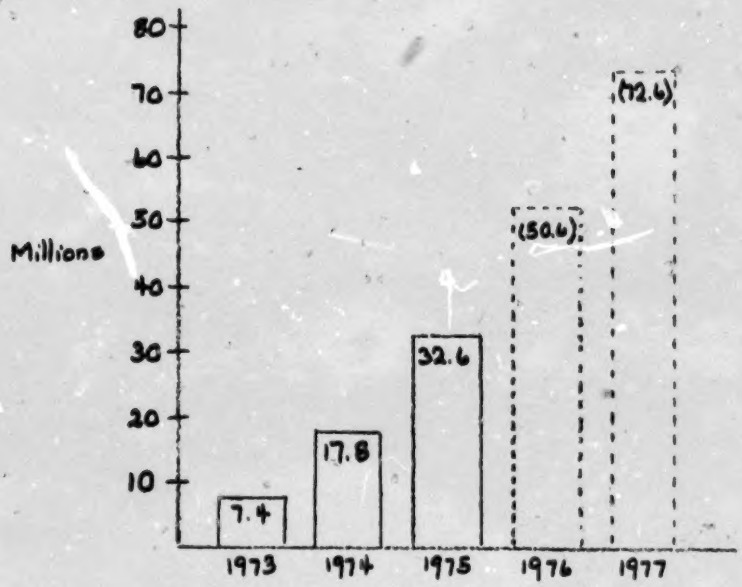
Hand-Held Calculator Sales Per Year
in the United States



That such estimated data are tenuous must be clearly understood. To illustrate this, Usiskin and Bell (1976) cite data "from a 1975 survey for industry by a reliable private research organization" (p. 2). Their data indicate sales of calculators to have been 8.34 million in 1973, 10.52 million in 1974, and 12.77 million in 1975, with a projection of 14.75 million in 1976. Another figure is provided by Electronics (8 January 1976), indicating that estimated sales are 21 million for 1976.* It is safe to say that the number of calculators being sold is large, increasing each year, and neither fully tabulated nor predictable!

The second table developed by Shumway presents the data in terms of cumulative sales estimates:

Cumulative Hand-Held Calculator Sales
ir the United States



* Shumway provided sales statistics for the top five leading magazines in 1974 (included in the 1975 Information Please Almanac), to serve as a referent for the magnitude of the figures:

TV Guide	20.1 million	Family Circle	8.3 million
Readers' Digest	18.8 million	Woman's Day	7.9 million
National Geographic	8.8 million		

The data provided by Usiskin and Bell (1976, p. 2) are slightly more conservative. Electronics (8 January 1976) indicated that "sales of handheld calculators have just about leveled in the U. S. . . . The balance of production for all types of calcu'ators has again shifted to Japan" (p. 86). Nevertheless, they predict that 1976 should be a "solid year" for calculators.

Shumway reported that calculator sales in over half the instances are being made to housewives and students for home and school use. However, it is frequently noted in marketing reports that the actual users of calculators (especially the simpler ones) are not clearly determined; it is particularly unclear how many of the instruments are actually in the hands of students. As one visits schools and talks with teachers, it is clear that this number is increasing. Shumway also noted that more and more sophisticated models are being purchased each year (especially as cost has dropped and as familiarity increases).

Prices

There are two price ranges which seem appropriate for school use:

(1) basic four- or five-function machines with limited memory

(2) fairly sophisticated scientific calculators with functions such as x^2, $\sqrt{x}$, 1/x, x!, $\sqrt[x]{y}$, y^x, log, e^x, arc, sin, cos, tan, sin h, degrees/radians, etc.

Examples of each of these include:

(1) basic four- or five-function machines	-	$10 - $20
e.g., Novus 650		$ 9.95 ($6.99)*
Sharp EL-8005		$19.95 ($13.95)
Texas Instruments TI-1200		$19.95 ($13.95)
Rockwell 10-R		($12.66)

* Prices in parentheses were observed in Columbus, Ohio in August 1975.

(2) scientific calculators	$50 - $100	
e.g., Melcor SC-635	$69.95	
Hewlett-Packard HP-21	$125.00	
Casio fx-11	$49.95	
Texas Instruments SR-50A	$99.95	($66.95)

Electronics (8 January 1976) indicated that sales of four-function, personal calculators are or will be:

1974 $265 million

1975 $268 million

1976 $276 million

1977 $301 million

In his analysis of the situation in August 1975, Shumway stated that the predicted retail sales price within one year for basic four- or five-function machines is $5. The predicted retail sales price within two years for scientific calculators is $25. He noted that most manufacturers would only make such predictions verbally. Others, however, have indicated that they believe that "rock bottom" has been reached, and that prices will either stabalize at current (February 1976) figures or begin to climbslightly for basic four-function calculators; prices for scientific calculators may, however, continue to decline (at least to some extent). This belief is based on the point that the production/marketing factors which combined to drive prices downward in 1975 parallel the situation of several years ago with television sets: prices now have nowhere to go but up, for basic four-function calculators. Refinements are still being made in the production of scientific calculators, however; for instance, one company recently placed a calculator on the market for $395, approximately half the cost of a comparable calculator from another company.

Perhaps the most pertinent point for educators is that it seems that

prices of calculators appropriate for use in elementary and secondary schools will not vary so greatly that there is reason either to speed or procrastinate in purchasing calculators on economic grounds.

Current Level of Use in Schools

Estimates of the extent to which calculators are now being used in schools are highly subjective. The project survey of school personnel suggested that about two-thirds of the schools used calculators in some way: this figure, however, is biased, since many schools were identified which were known to have been using calculators. Meiring's (1975) survey of Ohio secondary schools suggested that about 18% of the schools responding had systematic use of calculators. Most of the use occurred in grades 11 and 12. Thirty-four per cent report that calculators were prohibited in some classes or for certain classroom activities (notably tests). In the past six months (since these data were collected), it would appear that more teachers and schools have become involved in using the calculator in some way: perhaps an estimate in the range of 25% to 50% is the best projection that can be made.

Estimates of Future Use

Given the data on prices, the data on the number sold, and the fact that in the project survey teacher educators, state supervisors, and textbook publishers appear to be "sold" on the desirability of the use of calculators in schools, school use of calculators is likely to increase. Factors which might tend to control the use are parent and teacher reluctance to allow calculators in the classroom. As teachers and parents use calculators themselves, their reluctance is likely to diminish. By 1977, there could be widespread use of calculators in schools; by 1979, this will almost certainly be true.

III. The Cases For and Against Using Calculators in Schools

A primary purpose of this project was to identify the positions which educators hold regarding the use of calculators in elementary and secondary schools. When the responses on the questionnaires sent to school personnel, state supervisors of mathematics, and mathematics educators are analyzed, it becomes apparent that there is much similarity in their viewpoints. And in addition, a redundancy is apparent in many published articles, for these same points are reflected over and over.

The most frequently cited reasons for using calculators in schools are:

(1) They aid in computation. They are practical, convenient, and efficient. They remove drudgery and save time on tedious calculation. They are less frustrating, especially for low achievers. They encourage speed and accuracy.

(2) They facilitate understanding and concept development.

(3) They lessen the need for memorization, especially as they reinforce basic facts and concepts with immediate feedback. They encourage estimation, approximation, and verification.

(4) They help in problem solving. Problems can be more realistic and the scope of problem solving can be enlarged.

(5) They motivate. They encourage curiosity, positive attitudes, and independence.

(6) They aid in exploring, understanding, and learning algorithmic processes.

(7) They encourage discovery, exploration, and creativity.

(8) They exist. They are here to stay in the "real world", so we cannot ignore them.

The last reason -- the pragmatic fact that they exist and that they are appearing in the hands of increasing numbers of students -- is perhaps the most compelling. How they can be used to facilitate each of the other seven beliefs is therefore a question that must be attacked.

The most frequently cited reasons for not using calculators in schools are that:

(1) They could be used as substitutes for developing computational skills: students may not be motivated to master basic facts and algorithms.

(2) They are not available to all students. Because they cannot afford a calculator, some students are at a disadvantage.

(3) They may give a false impression of what mathematics is. Mathematics may be equated to computation, performed without thinking. Emphasis is on the product rather than on the process; structure is deemphasized. Mental laziness and too much dependence are encouraged; lack of understanding is promoted. Some students and teachers will misuse them.

(4) They are faddish. There is little planning or research.

(5) They lead to maintenance and security problems.

The first concern -- that students will not learn basic mathematical skills -- is one expressed most frequently by parents and by other members of the lay public, as reflected (and created) by newspaper articles. But it builds a strawman, for few educators believe that children should use calculators in place of learning basic mathematical skills. Rather, there is a strong belief that calculators can help children to develop and learn more mathematical skills and ideas than is possible without the use of calculators. Much serious attention must be given

by teachers and others to proving that this belief can be implemented and become fact.

For the Interim Report, Shumway considered the responses which educators gave on the questionnaires as reasons for and against using calculators, comments made in interviews and discussions with various persons, and statements in articles and other published literature. He then developed a section in which he expanded on the arguments for and against using calculators. Shumway's analysis is presented on the following pages (14-23).

Arguments for Using Calculators

The extreme of the point of view favoring the use of calculators is represented by this statement:

> "Hand-held calculators as sophisticated as the so-called 'scientific calculator' should be made readily available to all children, for all school work, from first grade on."

In support of such a position, variations of the following arguments may be cited:

(1) "There will no longer be any need for the usual paper-and-pencil algorithms for the basic operations. Algorithms are designed to carry out repetitive calculations efficiently, accurately, and without thinking. Clearly, the calculator is the best calculational algorithm available today. Paper-and-pencil algorithms might be taught for historical, cultural, or pedagogical purposes; however, few children (or adults) will choose paper-and-pencil algorithms when calculators are available."

(2) "Scientific calculators will not be expensive. The price of scientific calculators began only a few years ago at $400; currently, they are available for as little as $50. There is no reason to believe that they will not soon be available for less than $20 (which is the cost of two tanks of gas for a car). Cost will not significantly deter the widespread use of hand-held calculators."

(3) "Most children will probably learn the basic addition, subtraction, multiplication, and division facts in order to make estimations and to save time. Extensive drill and practice exercises will be unnecessary."

(4) "Decimals and scientific notation will be introduced early in first grade. Children will work with numbers such as .0285714285 and 1.893456 08. The first number is that part of a cake each of 35 children would get if it is divided into 35 equal parts; the second is the number of seconds a six-year-old child has lived. Children can and will work comfortably with such numbers. Calculators will facilitate early continuous experiences with a whole new class of numbers."

(5) "Mathematical exercises will be more realistic. Exercises will no longer have to be chosen so that there are integer solutions. So-called 'grubby numbers' and tedious calculations will be done with ease."

(6) "Calculators are fun. The motivational aspects of the hand-held calculator are exciting. Children create their own interesting problems. Low achievers generate new enthusiasm for mathematics because they finally have no fear of being unable to perform the necessary calculations. Children are eager to do mathematics when calculators are available."

(7) "The addition and multiplication algorithms for fractions can be delayed until algebra."

(8) "The calculator facilitates number sense. Because of their simplicity and speed, hand-held calculators will allow children to explore products, sums, powers, logarithms, trigonometric functions, etc., with numbers of all sizes with a frequency never before possible. Intuitive number sense will be much facilitated by such extensive, continuous, and early experience with numbers and their properties."

(9) "Hand-held calculators make calculations easy and practical for all children. It must be remembered that decimal notation, Arabic numerals, zero, paper-and-pencil algorithms, etc., were introduced not to teach mathematics, but to make calculations easier. The hand-held calculator was invented for the same reason."

(10) "Hand-held calculators stimulate interest in and facilitate the teaching of mathematical concepts. Homomorphic properties of functions, properties of logarithmic and exponential functions, characteristics of rational exponents, compounding continuous interest, combinatorics, trigonometric functions, limits, number theory, etc., can all be learned in more interesting manners because of the calculational power which the calculator provides."

(11) "The calculator can be used to facilitate problem solving. Open exploration and new problems can be offered to children because of the facilitating calculational power which the calculator provides. For example. learning to predict for what integer values of x will 1/x fill the calculator display screen teaches a great deal about our base ten numeration system and relative primes."

(12) "Hand-held calculators provide experience with the only practical algorithm which is used in society today. No business or profession carries out extensive calculations without the use of a calculator. Most family financial calculations will soon be done by calculator."

(13) "Hand-held calculators will place the emphasis on when and what operation to use rather than on how to perform the paper-and-pencil algorithm correctly."

(14) "There will be more interest in estimation. Since calculator errors tend to be dramatic rather than minute, estimating 'ballpark' answers will be useful in avoiding errors."

(15) "The power of mathematics used by the common man will increase astronomically. A simple example can be used to illustrate this. Suppose it takes $10,000 per year for a particular couple to retire today. Assuming an annual inflation rate of 5 per cent, how much per year would be required 20 years from today? The sequence (1.05, y^x, 20, x, 10,000, =) gives the answer of $26,533 in 10 seconds. Tailor-made family financial planning would be much improved by such calculational power."

(16) "More time will be available to teach mathematics in depth. Since calculators increase the speed and accuracy with which children can do calculations, much more time will be available to learn the concepts and principles of mathematics."

(17) "New topics in mathematics can be introduced into the curriculum. The calculational power of the calculator allows the consideration of new topics while the de-emphasis of paper-and-pencil algorithms produces more time for new topics."

Arguments Against Using Calculators

The extreme of the point of view against the use of calculators is represented by this statement:

> "Hand-held calculators should be banned from classroom use for mathematics."

Arguments cited in support of this position include variations of these points:

(1) "Hand-held calculators would destroy all motivation for learning the basic facts. Calculators do not remove the need to know basic facts such as 9 x 7. To raise children to run to their calculators for every simple calculation would be folly. Such dependence on calculators would be most unfortunate."

(2) "The use of calculators would destroy the basic, mainstream mathematics of the elementary-school curriculum. Society's major objective for elementary-school mathematics is that children learn the basic facts and be able to perform the paper-

and pencil algorithms for addition, subtraction, multiplication, and division. If calculators are allowed in schools, children will no longer see any need for basic calculational skills. Even banning calculators on certain days or only using them for checking would seem unfair and illogical to children. Calculators must not be used for any teaching of mathematics."

(3) "The cost of calculators prohibits their use. Schools simply cannot afford to provide calculators for children. The cost of hand-held calculators is prohibitive and their attractiveness makes them disappear all too frequently."

(4) "Calculators are particularly inappropriate for slow learners. What possible motivation would such children generate for learning an algorithm they know they can do on a calculator much more quickly and accurately? Calculators would insure that poorly motivated students would not learn the basic skills."

(5) "The child's notion of the nature of mathematics would be changed by the use of calculators. There is a real danger that if calculators are used, children will think that pushing buttons on a black box is mathematics."

(6) "The use of calculators would reduce children's ability to detect errors. We are all familiar with the belief that if a calculation was done on a calculator it must be right. Not only is such faith unjustified, but discovering errors of key-punching a calculator is almost impossible since there is no record of what was done."

(7) "Paper-and-pencil algorithms are still necessary, basic skills. Calculators can never be everywhere. Children must still be able to calculate on their own. The availability of calculators in schools would remove children's need for practicing the basic skills. Homework done at home would no longer ensure facility with the basic skills, since the home is likely to have a calculator. Schools must ban the use of calculators to ensure facility with the basic skills of arithmetic."

(8) "Batteries lose their charge and wear out. Dependency on batteries for computational arithmetic would be foolish."

(9) "The use of hand-held calculators would discourage mathematical thinking. If children can do any mathematical calculation by pressing a few buttons, problem solving will be done by guessing, not mathematical thinking. Try this, try that, keep doing things with the numbers until the answer looks right. Non-thinking guessing will become rampant if calculators are available in schools."

(10) "Parents are unalterably opposed to the use of calculators in the schools. The schools have failed miserably in the teaching of basic skills as it is. The introduction of calculators would be, in effect, not teaching mathematics at all. Schools would be exhibiting extreme political ineptness to introduce calculators."

Discussion of Arguments

It is probably obvious that the validity of the arguments in some of the statements above is questionable -- depending, to some extent, on your own viewpoint! It should also be obvious that not all proponents or opponents of using calculators in schools take extreme positions such as those identified. The following compromises have been suggested, for instance:

(1) Restrict the capability of the calculator through masking or making electronic modifications to control the operations available so that paper-and-pencil algorithms are still necessary and/or so that mathematical capabilities beyond the current curriculum are unavailable to the student.

(2) Restrict the use of calculators to checking answers only, or restrict the use to certain days of the week so that basic facts and paper-and-pencil algorithms are still necessary.

(3) Restrict the use of calculators to the upper grades (10-12), where presumably students have already learned the basic facts and the paper-and-pencil algorithms.

Some react to such compromises as an unworkable effort to "have your cake and eat it too", while others view them as examples of democratic compromise to achieve the best solution. The compromises do serve to focus attention on what appear to be the fundamental arguments regarding the hand-held calculator. The proponents' argument is essentially:

> "The hand-held calculator is the tool used in society today for calculations. Schools are 'burying their heads in sand' if calculators are not recognized and used as the calculational tool that they are."

The opponents' argument is essentially:

> "The principal objectives of mathematics instruction (at least in grades k-9) are that children learn the basic facts and the paper-and-pencil algorithms. Such learning will not occur if calculators are made available in schools."

It would seem that a rational approach to the resolution of the problem

(perhaps over-simplified) would involve:

(1) Determining current and future societal needs for the basic facts and the paper-and-pencil algorithms.

(a) If there are no needs for such skills, drop the emphasis on them and introduce the widespread use of calculators.

(b) If there are needs for such skills, move to question 2.

(2) Can the calculator be used in the classroom and still build students' needed skills (as identified in 1b)?

Such a procedure would seem to satisfy the concerns of opponents of the use of calculators. The proponents of the use of calculators would (probably) claim that such an over-simplification of the benefits of the use of calculators is ignoring a potentially powerful educational device.

Potential Implications of the Widespread Use of Calculators

Suppose we adopt the position that there will be widespread use of the calculator in schools. What are some of the benefits and disadvantages os such widespread use?

(1) Curriculum concerns

As Pollak (1976) has aptly described in his position paper (see Appendix D), there are two partial orderings which are often used in the designing of a mathematics curriculum. For example, the mathematical development of the number systems suggests that children ought to work with addition of whole numbers before they study addition of decimals. The algorithms for addition of decimals require facility with the addition of whole numbers. Hence the order: whole numbers, then decimals, rather than decimals followed by whole numbers. Such partial orderings may be called content orderings.

A second partial ordering which must be considered in curriculum development is a social value ordering. Topics in a mathematics

curriculum are included and ordered by the topic's worth to society. For example, the quadratic formula is included in the curriculum before compound interest because society views the quadratic formula as more essential for the needs of society than compound interest. Mathematically, either topic could be introduced before the other. Socially, the quadratic formula has priority over compound interest. Such partial orderings may be called societal orderings.

If the introduction of a new device such as the hand-held calculator makes a significant change in either the content ordering or the societal ordering, then major curriculum modification would seem appropriate. Such changes in the partial orderings are possible with the calculator. For example, the algorithm for addition of decimals is the same as the algorithm for addition of whole numbers: the same buttons are pushed for either. Thus, it may no longer be necessary or desirable to delay decimals until fifth grade.

A careful, extensive study of the impact of the calculator on the curriculum is needed: there appear to be significant changes which could (or ought to) be made. In their position paper, Usiskin and Bell (1976) (see Appendix F) present some initial suggestions on this task.

(2) Computational skills

The principle purpose of a calculator is to make calculations easy. Consequently all the basic operations of arithmetic, square roots, trigonometric functions, logarithms, etc., can be computed by very young children. Decisions regarding curriculum need no longer be made based on whether or not children can perform the

calculations, but rather on whether they understand the concepts involved. It is probably true that the concept of square root is easier to understand than the former computational algorithms for finding square roots. It may or may not be desirable to introduce square roots much earlier in the curriculum. The point is that the decision need not be based on the difficulty of teaching paper-and-pencil algorithms for finding square roots.

Compound calculations need no longer be avoided. "How many seconds have I been living?" may be a very reasonable calculation problem for a nine-year-old (60 x 60 x 24 x 365.25 x 9). The cost per gram of various candy bars is calculationally trivial and a reasonable question to pose. There no longer need be a fear of non-integral numbers (2.7 x 5 x 17.6 is as easy as 3 x 5 x16). Problems do not need to be artificially simplified. The numbers can be realistic.

At the minimum, one would expect some de-emphasis of the paper-and-pencil algorithms. Most calculations, in reality, will not be carried out by paper-and-pencil. It is likely that schools will begin teaching paper-and-pencil algorithms as another way to do calculations, but not the principal way.

(3) Teacher education

It is easy for the teacher educator to advocate widespread use of calculators. It is another matter for the classroom teacher actually to implement their use. The first difficulty encountered is parental opposition to calculators. The second difficulty is that the current curriculum is not designed for calculators. Exercises and problems which use the calculational power now available must currently be developed by the teacher. (Some

of the articles and books cited in Appendix A contain helpful suggestions; the position paper by Immerzeel, Ockenga, and Tarr (1976) in Appendix C provides a variety of specific activities; textbook publishers plan to have some materials available by 1977.)

The third difficulty is that most teachers lack the mathematical background necessary to deal with the questions and mathematics which can be generated by the use of calculators. For example, there is a mathematically honest explanation for the sine function which can be given to first graders. Most first-grade teachers would be unable to provide such an explanation. Many junior high school teachers would be unable to provide such an explanation. And many high school teachers would have difficulty with the hyperbolic sine function.

The fourth difficulty is that techniques for teaching mathematics with calculators have not been illustrated. The effect of calculators on children's number sense and other mathematical factors is not known, either through research or tradition. Each teacher must break new ground in the interaction of calculators and children learning mathematics.

(4) Budgets

Consider a typical elementary school with two classes at each grade level. In order to provide approximately one basic four-function calculator for each two students, such a school would need to spend $1800 ($10 per calculator for 180 calculators, or 15 per class). Consider such a cost in perspective: $1800 is the cost of 30 filmstrips or 120 minutes of 16 mm film. Given the impact on students, such a cost could be defended easily. Of

course, if one acquires scientific calculators, the cost would be $9000, or the equivalent of 150 filmstrips or 600 minutes of 16 mm film. It would appear that the widespread purchase of calculators may not be a major financial burden for a school which routinely purchases materials such as filmstrips or films.

The data on purchases of calculators suggest that many children will soon have access to a calculator regardless of the school action. The school's responsibility will probably be to have machines available for those children who do not have access to a calculator. The cost of such a requirement could soon be relatively low.

Summary

The impact of widespread use of hand-held calculators is likely to be:

(1) A de-emphasis on paper-and-pencil algorithms.

(2) More significant and interesting mathematics in the curriculum.

(3) Consumers and decision-makers much better prepared to deal with the voluminous amount of data in communications today.

Shumway's point of view expressed above is echoed to some extent in the position papers in Appendices C through F. Each of the writers of those papers expresses additional concerns and thoughtful comments; attention is directed particularly to the papers by Pollak and by Usiskin and Bell.

IV. Ways in Which Calculators Are Now Used in Schools

Calculators are being used in various ways in classrooms scattered throughout the country. Last year, such activities were generally more extensive at the secondary-school level; this year, elementary-school teachers are increasingly introducing them for specified purposes. They are recognizing that the calculator is a part of children's lives. In many instances (perhaps too many), use of the calculator is restricted to checking the result of paper-and-pencil computation. As teachers explore potential uses, and as more specific suggestions appear in print, additional use is made of the calculator.

Two fears must be expressed:

(1) That calculators will not be used appropriately, so that few positive benefits of their use are apparent.

(2) That teachers will indiscriminately buy materials for use with calculators (as in some cases they have done with metric materials).

As Immerzeel, Ockenga, and Tarr (1976) point out, to avoid "future shock" immaginative software must be developed. They also make recommendations regarding use of calculators (p. 5):

(1) Primary level: incidental use, especially in an interest corner.

(2) Intermediate level: availability in the school of class sets for occasional use

(3) Junior high level: availability of class sets for each teacher

(4) Senior high level: a calculator for every student, available anytime

They go on to provide a variety of specific illustrations for using the calculator at each of these levels, usually within the existing curriculum.

In general, certain patterns of use are evident:

(1) The district or school purchases a small number of calculators, which are given to teachers for exploratory activities. This is followed by discussion and decision on whether the district or school should purchase more (e.g., Columbus, Ohio).

(2) Remedial mathematics or Title I classes receive calculators for use with low achievers who have not previously learned computational skills well (e.g., Washington Irving High School, New York; Berkeley, California).

(3) Calculators are placed in advanced science and mathematics classes in secondary schools (e.g., Lubbock, Texas).

(4) Exploratory work on topics for which calculators seem most appropriate is in progress (e.g., under the direction of such mathematics educators as Immerzeel, Kessner, Rudnick, Scandura, Weaver).

(5) Pilot studies and/or research is being conducted on the effect of use of calculators (e.g., with low achievers at the secondary level in Chicago; in such California schools as Cupertino, Garden Grove, Los Angeles, San Diego, San Francisco, and Santa Barbara).

Among the variety of other activities, surveys of the attitudes of teachers toward calculators and/or uses being made of calculators have been conducted (e.g., Philadelphia; Shawnee Mission, Kansas; Ohio; California); these sometimes lead to the development of policy statements. The 1975 Annual Leadership Conference at the University of Michigan focused on the role of calculators, as did several groups at a September 1975 meeting on secondary school mathematics attended by educators from throughout Ohio. Workshops were presented at local, regional, and national

mathematics meetings in 1975, and plans are underway for extending such offerings during 1976 (e.g., the NCTM Name-of-Site Meeting in Detroit featured calculator workshops). The National Council of Teachers of Mathematics is developing film materials on calculators (with a grant from a calculator manufacturer). In cooperation with ERIC/SMEAC, NCTM is developing a compilation of teacher-suggested activities for use with calculators. Several journals (e.g., Instructor and Arithmetic Teacher) will have 1976 issues focused on the calculator.

Analysis of the published articles and books cited in Appendix A indicates that many fall into one of four categories:

(1) General statements about calculators: e.g., Denman, 1974; Higgins, 1974.

(2) Pros and cons: e.g., Etlinger, 1974; Fiske, 1975.

(3) Costs and features: e.g., Jesson and Kurley, 1975; Consumer Reports.

(4) Varied uses: e.g., Engel, 1974; Judd, 1975.

The annotations in Appendix A may provide a guide to locating materials of specific interest.

Usiskin and Bell (1976) (see Appendix F) take exception to merely incorporating calculator uses into the existing curriculum. For reasons which they state, "It is thus our belief that the insertion of calculators into K-6 classrooms using most existing curricula is fraught with peril" (p. 36). They argue for an alternative curriculum, and provide an assessment of how the curriculum may be restructured. They note that this may be threatening to those who view the present curriculum as optimally logical and sequential: their specific suggestions (pp. 40-49) could, however, suggest to many elementary teachers a different way of considering the use of calculators in schools.

V. Empirical Evidence

At the time that the Interim Report on this project was prepared in August 1975, attention was called to three points:

(1) At this time, there is comparatively little evidence on the effect of the use of hand-held calculators in schools. Studies have been exploratory in nature, often with the support of a calculator manufacturer.

(2) That the calculator can be used to teach certain topics seems clear: that significant achievement gains will result is not clear. As might be expected, attitudes are reported to be generally positive.

(3) Not all of the research has focused on significant questions (some has remained unpublished for this reason). There is a definite need to establish priorities and attack the questions that can and should be answered by research.

As this Final Report is prepared in February 1976, the same three points can be reiterated. The research picture has not essentially changed.

As Weaver (1976) has pointed out in his paper on needed research (see Appendix E):

> . . . The very newness of calculators provides little of a research base upon which to build. . . . The extent of ongoing research is very difficult to assess; this also is true of the nature of that research. We are given hints from the brief progress reports released by some projects . . . but by and large we have precious little information--and none of it definitive--regarding the extent and nature of ongoing research. (p. 18)

He goes on to express the thought that "When the annotated listing [of research on mathematics education] for calendar year 1975 is compiled, more calculator investigations are bound to appear; but there still will be no plethora of such investigations reported" (p. 18). Very true:

there has been no plethora; research evidence remains scarce (perhaps the 1976 listing will contain more?).

There is so little research on the use of hand-held calculators, in fact, that there is some question if it is worthwhile to review it. Most of the studies barely meet criteria for being termed "research -- "action research", "preliminary study", "inquiry", "exploration" are the words which investigators use in the published reports. There are rumors of many studies going on: most of them turn out to be explorations to find out what can be done with calculators. Some of the "hardest" data come from studies conducted by calculator manufacturers; not surprisingly, these indicate that students (a) can use the calculator with a variety of content and (b) achieve well when using the calculator. Many schools are checking data on their own students to find out the effect of the use of calculators with their students: this is a highly appropriate activity -- providing it continues as new suggestions and materials for using the calculator appear. Schools must be wary of selecting options too quickly -- of deciding that this way of using calculators is effective and that way isn't -- before the range of options (that is, a diversity of material designed for calculator use) has been developed.

Over the years, several dozen studies have been conducted with desk calculators. There has been some thought that this research might provide some useful information which would be applicable to hand-held calculators. Alas, the studies are not all designed as well as they should have been. On Table 1, 21 of the studies with desk calculators are summarized; note should be made of the small sizes of samples, the short lengths of time, the limited purpose of many of the studies, the evident confounding of variables.

TABLE 1

REPORTS OF RESEARCH ON DESK CALCULATORS

Author	Date	Grade level	N	Length	Type of research	Calculator use	Findings
Advani	1972		18 pupils (EMRs, NIs)	6 months	action	checking	significant increase in achievement, positive attitudes
Beck	1960	4-6	"several classes"		action (no data)		students can use calculators, motivation improved
Betts	1937	6	13 pupils	8 weeks	exploratory	doing examples	increased achievement and interest
Broussard et al.	1969	7-9	(low achievers)				significant achievement gain, more tackled more mathematics
Buchman	1969	sec.	1185 schools (N.Y.)		survey	-	13% of schools had calculators
Cech	1970, 1972	9	4 classes (general mathematics)	7 weeks	experimental	checking	no gains in skills or attitudes (therefore must be used earlier, longer)
Durrance	1965	6-8	70 pupils	9 weeks			calculator-use had no effect except on reasoning in grade 7
Ellis and Corum	1969		2 classes (low achievers)		experimental		no significant difference in achievement; better attitudes
Fehr et al.	1956	5	8 classes	4-1/2 mos.	experimental	doing multiplication	calculator group was better on tests of computation and reasoning, but not overall mathematics achievement; attitudes were positive

TABLE 1 (Continued)

Author	Date	Grade level	N	Length	Type of research	Calculator use	Findings
Findley	1967	9		1 year		working (with 2 types of textbook)	calculator plus traditional textbook resulted in better achievement only on arithmetic fundamentals when compared with calculator plus calculator-specific textbook
Gaslin	1972, 1975	9	6 classes (general mathematics)		experimental	type of algorithm for operations with rational numbers	no significant differences in achievement, transfer, retention, attitude, or rate (therefore calculator algorithm is viable alternative to conventional algorithm with or without calculator)
Hohlfeld	1974	5	79 pupils			immediate feedback on multiplication	daily drill with calculators resulted in higher achievement than drill with paper/pencil or "regular activities"
Johnson	1971	7		1 unit	experimental	working, checking	differences between calculator and non-calculator groups found on both achievement and attitude measures
Keough and Burke	1969						calculator group had better achievement than control group
Ladd	1974	9	201 pupils (10 classes) (low achievers)		experimental	working,	no significant differences between calculator and control groups was found

TABLE 1 (Continued)

Author	Date	Grade level	N	Length	Type of research	Calculator use	Findings
Longstaff et al.	1968	5,9	2 groups				attitudes were positive
Mastbaum	1969	7-8	8 classes (slow learners)	6 months	experimental	working	calculator-use did not significantly improve attitude, or increase mathematics ability, non-calculator computational skill, mastery of concepts, or problem solving ability -- but no significant differences were found between groups
Schott	1955	4-9		4 months	experimental	solving problems	calculator group scored higher than control group; problem-solving ability increased
Shea	1974	4		30 weeks	experimental	flow charting	calculator group scored higher than non-calculator group on computation but not other achievement tests or attitude
Stocks	1972		15 pupils (EMRs)	15 lessons		working	EMRs capable of doing division with calculator; both groups improved
Triggs	1966	primary	1 class	9 weeks	action		both groups improved

There appear to be very few potentially transferable findings from the studies with desk calculators:

(1) Children can learn to use calculators.

(2) Children generally enjoy using calculators.

(3) Low achievers may profit from using calculators, but calculator use should not be restricted to low achievers.

(4) Calculators can be used for checking paper-and-pencil computation.

(5) Calculators may or may not facilitate particular types of achievement.

On Table 2, 8 studies on hand-held calculators are summarized. A few additional comments about each might aid in making readers further aware of the limitations of the research information on hand-held calculators.

Comments on Research Reports on Hand-held Calculators

The major goal of the study reported by Hawthorne and Sullivan (1975) was to "discover how (and if) the calculators could enrich, supplement, support, and motivate the regular program. There was no intent to change the program to fit the calculator" (p. 29). Barrett and Keefe (1974) expanded on some of the ways the students used the calculator: to check answers and in working with verbal problems, means, probability, palindromes, functions, and multiplication with decimals.

A comparison with a matched group indicated that the mean scores of students using calculators were higher ($p < .02$) on the concepts and computation sections of the test than were the corresponding scores for students not using calculators, but the two groups performed about equally well on the problem-solving section of the test. Two comments of interest

TABLE 2

REPORTS OF STUDIES WITH HAND-HELD CALCULATORS

Author	Date	Grade level	N	Length	Type of research	Calcultor use	Findings
Bitter and Nelson	1975	4-7			action	working, checking, games	calculator group achieved better than non-calculator group, had better attitudes
Hawthorne and Sullivan (Barrett and Keefe, 1974)	1975	6	96 pupils	1 year	action	working, checking	calculator groups higher on concepts and computation, not as high on problem solving; interest sustained
Kelley and Lansing	1975	7-8	8 classes	1 year	experimental	remedial	calculator group achieved higher than skill group on reasoning, computation
Kessner	1975	k,1			field test	working	children learn using calculators
Meiring	1975	7-12	111 schools (Ohio)	-	survey	-	calculators systematically used by 18.9%; 35.1% had guidelines; 34.2% prohibited calculator use
Schafer, Bell, and Crown	1975	5	5 classes (120 pupils)	2 days	preliminary study		calculator group scored significantly higher on calculator examples; no differences on non-calculator examples
Spencer	1975	5,6	84 pupils	8 weeks	experimental	doing computation worksheets	calculator group better than non-calculator group on reasoning in grade 5, on computation and total test in grade 6
Weaver	1976	2,3,5	7 classes	3 years	exploratory	operations, number sentences	pupils encounter no consequential problem using calculators

are made by Hawthorn and Sullivan:

> Project evaluators do not believe that calculators have any great inherent ability to support and motivate mathematical study, though these instruments definitely have some powerful computational capability. (p. 31) [Oh?]
>
> Perhaps another study can shed some light on what effect calculators have on learning mathematics if used by children without any particular direction by teachers. (p. 31) [Why?]

One wishes that there were some discussion of each of these points in the article.

Bitter and Nelson (1975) developed a "diagnostic remediation mathematics curriculum utilizing the hand-held calculator". A group using this curriculum was compared with a group using a commercial hand-held calculator mathematics remediation program and with two groups using the "normal" curriculum, one with calculators available and one without calculators. While no data were reported in the article, the authors note that " . . . all three calculator approaches provided for significant statistical gains in both the cognitive and attitudinal domain as opposed to the control group."

Analysis of data from Project Equip, a mini-calculator program for teaching mathematics sponsored by Berkeley schools, was reported by Kelley and Lansing (1975). Two seventh-grade and two eighth-grade mathematics classes for low achievers were involved. Neither experimental nor control classes showed statistically significant gains on the CTBS (the experimental group mean was 4.87 on the October pretest, 4.98 on the May posttest; the control group's respective scores were 5.29 and 5.30). On the CTBS computation subtest, the calculator group did significantly better, however (4.9 for the control group, 6.5 for the experimental group). And on the NLSMA Reasoning Test, the gain of the control group was significant at the .08 level (0.9 points); the calculator group gained 1.9 points, which was significant at the .001 level.

Kessner (1975) presented information on a primary mathematics project designed "to research, develop, and field test activities in which kindergarten and first-grade children can use hand-held electronic calculators to promote their mathematics learning". Simplified calculators are "coupled with gamelike modules" to teach the "complex aspects of counting and the operations of addition and subtraction". In the project information cited, no data are reported.

Schafer, Bell, and Crown (1975) report on a limited "inquiry" in which one group of fifth-grade pupils was given a pretest, and then given calculators for two (2) days, with problems to work and encouragement to ask questions about the calculator. A week later a posttest was given. Overall there were no significant differences. Those who had calculators, however, scored significantly higher on examples on which the calculator could be used, while there were no significant differences on non-calculator examples (although the score of the calculator group was lower than that of the non-calculator group). One wonders whether the same statistical result could be replicated: the conditions were obviously loose. The comments in Appendix A of the Usiskin and Bell position paper (1976) report further explorations with about 20 teachers. The work does not appear to be systematic, however: it is purely exploration to find out how pupils (and teachers) react.

Weaver (1976), on the other hand, has since 1973 been systematically exploring the use of various calculators at several grade levels, in ways which connect with his (and his students') previous research on mathematical sentences and properties of operations. Although he reports some empirical data, "the principal intent of the project to date has not been hypothesis formulation and testing," but informal exploration as a necessary stage to precede controlled experimentation. This "exploratory

work was independent of ongoing mathematics programs . . .", although this year "teachers are opting to have pupils use the calculator from time to time in connection with the ongoing mathematics programs". Among the emphases of the exploratory work have been chaining, doing and undoing, and related number sentences.

Spencer (1975) investigated the effect of using calculators on computation skill, reasoning ability, and total arithmetic achievement, as measured by the Iowa Test of Basic Skills. Forty pupils in grade 5 and 44 pupils in grade 6 were randomly assigned either to a group using calculators or to a group using paper and pencil without calculators. For eight weeks, both groups worked with computation worksheets prepared by the experimenter; unfortunately, the abstract of the study does not indicate the nature of these worksheets. At the fifth-grade level, the only significant difference found was on the reasoning test, favoring the calculator group. In grade 6, significant differences favored the calculator group on the computation test and on the total test; a tendency for the calculator group to have higher scores on the reasoning test was also noted.

The locations at which this research and development work is being conducted are diverse: Arizona, California, Illinois, Iowa, New York, Wisconsin. In other states, other projects are going on, with no published results as yet. For instance, Rudnick (Eye on Education, 1975) is currently directing a project in Pennsylvania with seventh graders to investigate the effects of the availability and use of the calculator on achievement and attitudes. Capoferi and Winowski (1975) present a design for a study to be conducted in Michigan schools. One hopes that carefully designed research will be planned at many other locations.

Needed Research

In his position paper in Appendix E, Weaver discusses some of the research questions which should have some priority. He called attention to the point that

> the greatest thing we have to fear today about the calculator vis-á-vis school mathematics curricula is the degree of fear that already exists about the calculator vis-á-vis school mathematics curricula. (p. 2)
>
> To many persons the calculator threatens to violate certain tenets regarding school mathematics learning and instruction--tenets that are adhered to more tenaciously than I might have expected. Suggestions for calculator uses are made within the constraints of those tenets . . . and any research that might be implicit in such suggestions would be similarly constrained.
>
> Some other persons, however, appear to be willing--possibly even anxious--to suggest calculator uses that may challenge certain of our cherished tenets. (p. 5)

Weaver distinguishes between three types of curricula -- calculator-assisted, calculator-modulated, and calculator-based -- and points out that "research should not be unmindful of such differential roles". After citing the research questions included in the NACOME Report (1975), he discusses six others for which answers should be sought. Each in turn can lead to a series of investigations.

Summary

Evidence on the effectiveness of calculator use is largely experiential. A concise summary of the suggestions for research which should be conducted to determine the potential and problems of calculator use in schools includes investigations related to:

- when and how to introduce calculators
- effective procedures for learning basic facts, computational skills, problem solving, and various mathematical ideas

- effective calculator algorithms
- long-range effects of using calculator algorithms
- need for paper-and-pencil algorithms
- effect of calculator use with specific content and curricula
- effect of curricula sequence/emphasis changes
- relationship between work with calculators and computers
- changes in teacher education curricula
- optimal calculator designs

One very specific caution must be emphasized: attempts at restructuring the curriculum, either extensively or minimally, must not proceed independently of research. The two are integrally interwoven, and one cannot be effective without the other.

VI. Some Recommendations

A variety of recommendations has been incorporated at many points in this Report. Many others are specified or suggested in each of the position papers. These are generally stated at the end of this section.

But first, some recommendations which have not been cited previously will be listed. These were given by educators in response to a question on several questionnaires in the survey conducted early in this project. They range from the general to those specific to the curriculum.

Recommendations from Educators Surveyed

1. Experiment and plan.
    a. Learn to use calculators yourself first, finding meaningful ways to use them.
    b. Use calculators with students only after considerable thought as to how, when, and why.
    c. Develop a school-wide policy and guidelines.
    d. Develop ways to incorporate calculators into the existing curriculum, and develop new curriculum as necessary.
    e. Plan a reasonable inservice program, evaluation, and research.
    f. Use in early grades with care, if at all.
2. Survey available calculator models carefully and buy good equipment, commensurate with student needs. Make sure that all students have access to a calculator.
3. Change teaching emphases to concept development, algorithmic processes, when to apply various operations, and problem solving using real-life and interdisciplinary applications.

4. Do not ignore the development of computational skill.
5. Think of calculators as a tool to extend mathematical understanding and learning by making traditional work easier. The focus can be on process because the product is assured.
6. Place more emphasis on problem-solving strategies. Use practical, realistic, significant problems, and more applications.
7. Spend less time on computational drill, more time on concepts and the meaning of operations. Use more laboratory activities where computation is involved but the emphasis is on learning mathematical concepts. Decrease the use of tedious, complicated algorithms; emphasize algorithmic learning, including student development of algorithms.
8. De-emphasize fractions, and emphasize decimals, introducing them earlier.
9. Emphasize estimation and approximation (including mental computation skills), checking and feedback, exploration and discovery.
10. Do more and/or earlier work with such ideas as place value, the decimal system, number theory, number patterns, sequences, limits, functions, iteration, statistics, probability, flow charting, computer literacy, large numbers, negative numbers, scientific notation, data generation, and formula testing.

Two points should be made in connection with the above recommendations:

(1) There was not consensus on all of them, nor were they all cited with equal frequency. A selection process occurred, which may reflect the beliefs of the author of this Report.

(2) The overlap of the recommendations with statements in other published materials is evident.

Major Recommendations

1. A thorough analysis of the mathematics and other appropriate curricula of elementary and secondary schools should be conducted to determine:
   a. how calculator use could be optimally integrated with existing curricula (see Section IV, Immerzeel et al.).
   b. how curricula should be revised/redeveloped to incorporate optimal use of calculators (see Section III, Pollak, Usiskin and Bell).
2. A careful plan for systematic research should be developed (see Section V and Weaver).
3. Following the above steps, appropriate research related to, and development of, curricula should be initiated.
4. Experiences for teachers at both inservice and preservice levels should be provided, to aid them in using calculators with students.
5. Information about research and development efforts must be communicated (with speed and accuracy) to parents and other non-educators, as well as to educators.

These recommendations are based on the assumption, derived from analysis of information secured during the project, that calculators are increasingly being accepted as an instructional tool (by both teachers and parents). Therefore an immediate need exists for sound and substantial research and development efforts.

References

Advani, Kan. The Effect of the Use of Desk Calculators on Achievement and Attitude of Children with Learning and Behaviour Problems. A Research Report. December 1972. ERIC: ED 077 160.

Barrett, Ray and Keefe, Michael. Using Hand-Held Calculators in Sixth Grade Classes. Albany, New York: The State Education Department, Bureau of Mathematics Education, 1974.

Beck, Lois L. A Report on the Use of Calculators. Arithmetic Teacher 7: 103; February 1960.

Betts, Emmett A. A Preliminary Investigation of the Value of a Calculating Machine for Arithmetic Instruction. Education 58: 229-235; December 1937.

Bitter, Gary G. and Nelson, Dennis. Arizona Migrant Education Hand-Held Calculator Project. Migrant Educator 1: 1-3; 1975.

Broussard, Vernon; Fields, Albert; and Reusswig, James. A Comprehensive Mathematics Program. AV Instruction 14: 43-44; February 1969.

Buchman, Aaron L. The Use of Calculators and Computers in Mathematics Instruction in New York State High Schools. School Science and Mathematics 69: 385-392; May 1969.

Capoferi, Alfred and Winowski, Eugene. Macomb Intermediate School District, Van Dyke Schools Cooperative Project - Exploration of Classroom Use of the Hand Calculator in Grades 4-6. Mt. Clemens, Michigan: Macomb Intermediate School District, 1975.

Cech, Joseph Philip. The Effect the Use of Desk Calculators Has on Attitude and Achievement in Ninth-Grade General Mathematics Classes. (Indiana University, 1970.) Dissertation Abstracts International 31A: 2784; December 1970. (See also ERIC: ED 041 757.)

Cech, Joseph P. The Effect of the Use of Desk Calculators on Attitude and Achievement with Low-Achieving Ninth Graders. Mathematics Teacher 65: 183-186; February 1972.

Denman, Theresa. Calculators in Class. Instructor 83: 56-57; February 1974.

Durrance, Victor Rodney. The Effect of the Rotary Calculator on Arithmetic Achievement in Grades Six, Seven, and Eight. (George Peabody College for Teachers, 1964.) Dissertation Abstracts 25: 6307; May 1965.

Ellis, June and Corum, Al. Functions of the Calculator in the Mathematics Laboratory for Low Achievers. 1969. ERIC: ED 040 847.

Engel, C. William. Meet Minnie Calculator! In Fostering Creativity Through Mathematics (edited by Betty K. Lichtenberg and Andria P. Troutman). Tampa: Florida Council of Teachers of Mathematics, 1974. Pp. 69-77.

Etlinger, Leonard. The Electronic Calculator: A New Trend in School Mathematics. Educational Technology 14: 43-45; December 1974.

Fehr, Howard F.; McMeen, George; and Sobel, Max. Using Hand-Operated Computing Machines in Learning Arithmetic. Arithmetic Teacher 3: 145-150; October 1957.

Findley, Robert Earl. An Evaluation of the Effectiveness of a Textbook, Advanced General Math, Used by Ninth Grade General Mathematics Classes. (Colorado State College, 1966.) Dissertation Abstracts 27A: 2440-2441; February 1967.

Fiske, Edward B. Educators Feel that Calculators Have Both Pluses and Minuses. The New York Times, Section IV, p. 7; January 5, 1975.

Gaslin, William Lee. A Comparison of Achievement and Attitudes of Students Using Conventional or Calculator-Based Algorithms for Operations on Positive Rational Numbers in Ninth-Grade General Mathematics. Dissertation Abstracts International 33A: 2217; November 1972.

Gaslin, William L. A Comparison of Achievement and Attitudes of Students Using Conventional or Calculator-Based Algorithms for Operations on Positive Rational Numbers in Ninth-Grade General Mathematics. Journal for Research in Mathematics Education 6: 95-108; March 1975.

Hawthorne, Frank S. and Sullivan, John J. Using Hand-held Calculators in Sixth-Grade Mathematics Lessons. New York State Mathematics Teachers' Journal 25: 29-31; January 1975.

Higgins, Jon L. Mathematics Programs Are Changing. Education Digest 40: 56-58; December 1974. (Reprint from NASSP Curriculum Report.)

Hohlfeld, Joseph Francis. Effectiveness of an Immediate Feedback Device for Learning Basic Multiplication Facts. (Indiana University, 1973.) Dissertation Abstracts International 34A: 4563; February 1974.

Jesson, David and Kurley, Frank. Specifications for Electronic Calculators. Mathematics Teaching 70: 42-43; March 1975.

Johnson, Randall Erland. The Effect of Activity Oriented Lessons on the Achievement and Attitudes of Seventh Grade Students in Mathematics. (University of Minnesota, 1970.) Dissertation Abstracts International 32A: 305; July 1971.

Judd, Wallace. A New Case for the Calculator. Learning 3: 41-48; March 1975.

Kelley, J. L. and Lansing, Ira. Interim Report on Project EQUIP. Berkeley, California, July 1975.

Keough, John J. and Burke, Gerald W. Utilizing an Electronic Calculator to Facilitate Instruction in Mathematics in the 11th and 12th Grades. Final Report. July 1969. ERIC: ED 037 345.

Kessner, Arthur. Exploring Calculators in Primary Mathematics. Berkeley, California: Regents of the University of California, 1975.

Ladd, Norman Elmer. The Effects of Electronic Calculators on Attitude and Achievement of Ninth Grade Low Achievers in Mathematics. (Southern Illinois University, 1973.) Dissertation Abstracts International 34A: 5589; March 1974.

Longstaff, F. R. et al. Desk Calculators in the Mathematics Classroom. June 1968. ERIC: ED 029 498.

Mastbaum, Sol. A Study of the Relative Effectiveness of Electric Calculators or Computational Skills Kits in the Teaching of Mathematics. (University of Minnesota, 1969.) Dissertation Abstracts International 30A: 2422-2423; December 1969.

Schafer, Pauline; Bell, Max S.; and Crown, Warren D. Calculators in Some Fifth-Grade Classrooms: A Preliminary Look. Elementary School Journal 76: 27-31; October 1975.

Schott, A. F. Adventure in Arithmetic. Educational Screen 34: 65-67; 1955.

Shea, James Francis. The Effects on Achievement and Attitude Among Fourth Grade Students Using Calculator Flow-Charting Instruction vs. Conventional Instruction in Arithmetic. (New York University, 1973.) Dissertation Abstracts International 34A: 7499; June 1974.

Spencer, Ann Nora. Using the Hand-held Calculator in Intermediate Grade Arithmetic Instruction. (Lehigh University, 1974.) Dissertation Abstracts International 35A: 7048-7049; May 1975.

Stocks, Sister Tina Marie. The Development of an Instructional System Which Incorporates the Use of an Electric Desk Calculator as an Aid to Teaching the Concept of Long Division to Educable Mentally Retarded Adolescents. (Columbia University, 1972.) Dissertation Abstracts International 33A: 1049-1050; September 1972.

Triggs, E. The Value of a Desk Calculating Machine in Primary School Mathematics. Educational Research 9: 71-73; November 1966.

Weaver, J. F. Calculator-influenced Explorations in School Mathematics: Number Sentences and Sentential Transformations I, II. Project Paper 76-1. Madison: Wisconsin Research and Development Center for Cognitive Learning, January 1976.

Calculators. Consumer Reports 40: 533-541; September 1975.

The Mini-Calculator Project. Eye on Education 4: 8-9; Number 1, 1975. (Temple University College of Education.)

Overview and Analysis of School Mathematics Grades K-12. Washington: Conference Board of the Mathematical Sciences, National Advisory Committee on Mathematical Education (NACOME), November 1975.

World Electronics Markets: Mounting a Push Toward a Slow Modest Recovery. Electronics 49: 83-106; January 8, 1976.